Hong Kong Mathematics Olympiad (2009 / 2010)

Final Event 1 (Group)

香港數學競賽 (2009 / 2010)

決賽項目 1 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求 $\sin^2 1° + \sin^2 2° + \cdots + \sin^2 89°$ 的值。

Find the value of $\sin^2 1° + \sin^2 2° + \cdots + \sin^2 89°$.

2. 已知 $\dfrac{x+z}{2z-x} = \dfrac{z+2y}{2x-z} = \dfrac{x}{y}$。求 $\dfrac{x}{y}$ 的值。

Given that $\dfrac{x+z}{2z-x} = \dfrac{z+2y}{2x-z} = \dfrac{x}{y}$. Find the value of $\dfrac{x}{y}$.

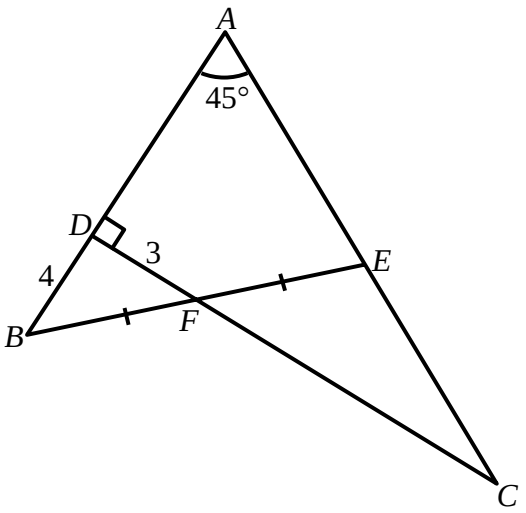
3. 求方程 $(2^x-4)^3 + (4^x-2)^3 = (4^x+2^x-6)^3$ 的所有實根 x 的總和。

Find the sum of all real roots x of the equation

$(2^x-4)^3 + (4^x-2)^3 = (4^x+2^x-6)^3$.

4. 在圖一，若 $AB \perp CD$，F 是 BE 的中點，$\angle A = 45°$，$DF = 3$，$BD = 4$ 及 $AD = n$，求 n 的值。

In Figure 1, if $AB \perp CD$, F is the midpoint of BE, $\angle A = 45°$, $DF = 3$, $BD = 4$ and $AD = n$, find the value of n.



圖一

Figure 1

Hong Kong Mathematics Olympiad (2009 / 2010)

Final Event 2 (Group)

香港數學競賽 (2009 / 2010)

決賽項目 2 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 $p=2-2^2-2^3-2^4-...-2^9-2^{10}+2^{11}$，求 p 的值。

If $p=2-2^2-2^3-2^4-...-2^9-2^{10}+2^{11}$, find the value of p.

2. 已知 x, y, z 為 3 個相異實數。若 $x+\frac{1}{y}=y+\frac{1}{z}=z+\frac{1}{x}$ 及 $m=x^2y^2z^2$。求 m 的值。

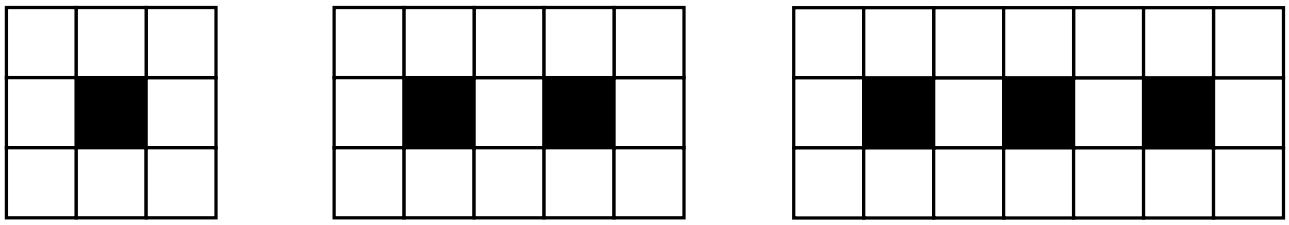
Given that x, y, z are three distinct real numbers. If $x+\frac{1}{y}=y+\frac{1}{z}=z+\frac{1}{x}$ and $m=x^2y^2z^2$, find the value of m.

3. 已知 x 為一正實數，且滿足 $x\cdot 3^x=3^{18}$。若 k 是一正整數且 $k<x<k+1$，求 k 的值。

Given that x is a positive real number and $x\cdot 3^x=3^{18}$. If k is a positive integer and $k<x<k+1$, find the value of k.

4. 圖一所示為利用黑白兩種顏色湊成有規律的圖形。求第 95 個圖形的白色格子的數目。

Figure 1 shows the sequence of figures that are made of squares of white and black.
Find the number of white squares in the 95th figure.



第 1 個
1st figure

第 2 個
2nd figure

第 3 個
3rd figure

圖一 Figure 1

Hong Kong Mathematics Olympiad (2009 / 2010)

Final Event 3 (Group)

香港數學競賽 (2009 / 2010)

決賽項目 3 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求 $101^{303}+303^{101}$ 的最小質因子。

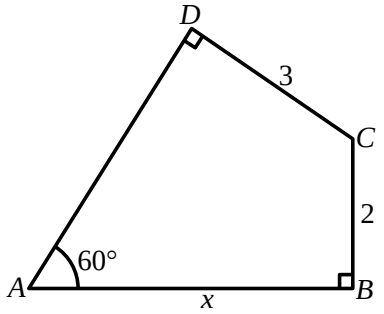
Find the smallest prime factor of $101^{303}+303^{101}$.

2. 設 n 為 $\dfrac{1}{\frac{1}{1980}+\frac{1}{1981}+\cdots+\frac{1}{2009}}$ 的整數部分，求 n 的值。

Let n be the integral part of $\dfrac{1}{\frac{1}{1980}+\frac{1}{1981}+\cdots+\frac{1}{2009}}$, find the value of n.

3. 在圖一中，若 $\angle A=60°$、$\angle B=\angle D=90°$、$BC=2$、$CD=3$ 及 $AB=x$，求 x 的值。

In Figure 1, if $\angle A=60°$, $\angle B=\angle D=90°$, $BC=2$, $CD=3$ and $AB=x$, find the value of x.



圖一 Figure 1

4. 已知函數 f 對所有實數 x 皆滿足 $f(2+x)=f(2-x)$，且 $f(x)=0$ 恰好有四個相異實根。求這四個相異實根之和。

Given that the function f satisfies $f(2+x)=f(2-x)$ for every real number x and that $f(x)=0$ has exactly four distinct real roots. Find the sum of these four distinct real roots.

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Final Event 4 (Group)
香港數學競賽 (2009 / 2010)
決賽項目 4 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知方程 $(a-1)x^2-mx+a=0$ 的兩根均為正整數。求 m 的值。

Given that the equation $(a-1)x^2-mx+a=0$ has two roots which are positive integers. Find the value of m.

2. 已知 x 為一實數及 $y=\sqrt{x^2-2x+2}+\sqrt{x^2-10x+34}$。求 y 的最小值。

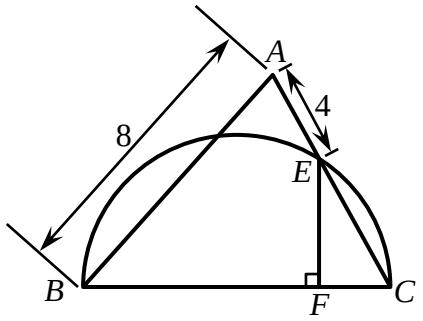
Given that x is a real number and $y=\sqrt{x^2-2x+2}+\sqrt{x^2-10x+34}$. Find the minimum value of y.

3. 已知 A、B、C 為正整數，且 A、B 和 C 的最大公因數等於 1。若 A、B、C 滿足 $A\log_{500}5+B\log_{500}2=C$，求 $A+B+C$ 的值。

Given that A, B, C are positive integers with their greatest common divisor equal to 1. If A, B, C satisfy $A\log_{500}5+B\log_{500}2=C$, find the value of $A+B+C$.

4. 在圖一中，BEC 是一半圓形及 F 是直徑 BC 上的一點。已知 $BF:FC=3:1$，$AB=8$ 及 $AE=4$。求 EC 的長度。

In Figure 1, BEC is a semicircle and F is a point on the diameter BC. Given that $BF:FC=3:1$, $AB=8$ and $AE=4$. Find the length of EC.



圖一 Figure 1